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Partial difference equation based model reference control of a multiagent network of underactuated aquatic vehicles with strongly nonlinear dynamics☆

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ABSTRACT

In a recent work, the authors presented an extension of robust model reference adaptive control (MRAC) laws for spatially varying partial differential equations (PDEs) proposed by them earlier for the decentralized adaptive control of heterogeneous multiagent networks with agent parameter uncertainty using the partial difference equations (PdEs) on graphs framework. The examples provided demonstrated the capabilities of this approach under the assumption that individual vehicles executing coordinated maneuvers were fully actuated and characterized by linear dynamics. However, detailed models for autonomous vehicles – whether terrestrial, aerial, or aquatic – are often underactuated and strongly nonlinear. Using this approach, but assuming the plant parameters to be known, this work presents the model reference (MR) control laws without adaptation for the coordination of underactuated aquatic vehicles modeled individually in terms of strongly nonlinear dynamic equations arising from ideal planar hydrodynamics. The case of unknown plant parameters for this class of underactuated agents with complex dynamics is an open problem. The paper is based on an invited talk on adaptive control presented at the 2008 World Congress of Nonlinear Analysts.

1. Introduction

The coordination of aquatic vehicles is a topic of growing interest in the context of applications ranging from military reconnaissance to environmental sampling [1]. However, an underwater vehicle that mimics locomotion of fish is an underactuated system, thereby posing additional challenges for designing control laws capable of enforcing its reasonable tracking capability. Tracking control of a single vehicle using backstepping is presented in [2] and references therein. However, designing a trajectory following control law for an underactuated aquatic vehicle typically requires lengthy algebra and the resulting control laws are rather involved. Control of a network of such vehicles presents additional difficulties.

The latter difficulty can be in part addressed by extending control laws designed for systems represented by partial differential equations to multiagent systems. This is accomplished using the framework of PdEs on graphs recently introduced in [3]. As indicated in [3], if parabolic or hyperbolic PDEs are made to exhibit a desirable dynamics (cf. [4] or [5]), PdEs on graphs defined through Laplacian operators essentially inherit this dynamics under similar conditions. The application of this setting to multiagent systems is carried out in [6] and [7]. Recently, using this framework the authors

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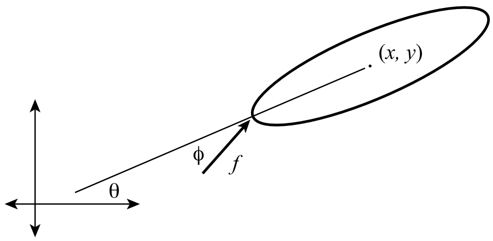

Fig. 1. Simplified model for an underwater vehicle.

presented in [8] an extension of robust MRAC laws for spatially varying PDEs under distributed sensing and actuation proposed in [9] and [10] to the decentralized adaptive control of heterogeneous multiagent networks with fully actuated agents under agent parameter uncertainty.

Using the approach of [8] and assuming no parameter uncertainty, this work presents the model reference (MR) control laws that address position tracking by means of a network of underactuated aquatic vehicles modeled in terms of ideal planar hydrodynamics. Instead of trajectory following, a reference model characterized by a simpler dynamics is specified, and the control objective is to steer the system to track the model. In [11] the reference model for systems with double-integrator dynamics was chosen to imitate the dynamics of an undamped wave equation, whereas in [12] the control law was designed so as to make the plant behave like a strongly damped wave equation. Here we choose a reference model that mimics the dynamics of a viscously damped wave equation.

The reference model generates a separate reference trajectory for each vehicle using a distinct reference input for each of them. It is reasonable to presume that only some of the vehicles, referred to as *leaders*, are equipped with sophisticated sensing instrumentation and hence are capable of determining their particular desired trajectories. The reference input for the *leaders* should hence be designed so that the reference trajectory generated by the reference model for the *leaders* corresponds to the desired trajectory. This design issue has been dealt with in detail in [8]. Hence in this work we assume directly that the reference inputs for the *leaders* are specified. The remaining vehicles are referred to as *followers*. The reference input for the followers is always set to zero. The reference trajectory for the followers is determined by those of the leaders through their coupling in the reference model. The coupling is chosen so that the reference trajectory of each follower is affected only by the reference trajectory of the neighboring vehicles. This ensures that the implementation of the control law involves communication between neighboring agents only, hence making the configuration decentralized.

2. Individual vehicle dynamics

2.1. Dynamics of a free elliptical body in an ideal fluid

In the present work each individual vehicle is modelled as an elliptical body moving in an otherwise quiescent infinite ideal fluid in the plane. The dynamics of such a body is described in many standard hydrodynamics texts, including [13], and is equivalent to the dynamics of a rigid planar body in space with an *effective mass* that depends on the body's direction of motion. An elliptical body, in particular, exhibits a larger effective mass for the lateral translation than for the longitudinal one. Control is introduced into the model in the form of an external force with magnitude f applied to the rearmost point on the body and a direction relative to the body's longitudinal axis specified by the angle ϕ, as shown in Fig. 1. The input force is defined relative to a body-fixed frame to represent the influence of a thruster or a propulsive appendage affixed to the body.

The position of the vehicle is characterized by the x, y-coordinates of its center of mass and a θ-coordinate specifying its orientation, as shown in Fig. 1. The equations of motion for the forced system may be obtained through straightforward application of the integral Lagrange–d'Alembert principle [14], defining the Lagrangian for the system as the total kinetic energy of the body and the surrounding fluid. If $2a$, $2b$, and ρ_e are the length, the width, and the density of the body, respectively, and ρ_f is the density of the fluid, then the principal effective inertias of the body are given by

$$\mu = ab\pi\rho_e + b^2\pi\rho_f, \qquad \nu = ab\pi\rho_e + a^2\pi\rho_f,$$

and

$$\iota = \frac{1}{4}ab(a^2+b^2)\pi\rho_e + (a^2-b^2)^2\rho_f.$$

Defining $\kappa = \mu + \nu$ and $\lambda = \mu - \nu$, the equations governing the dynamics of a single vehicle take a relatively compact form:

$$\begin{bmatrix} \kappa\ddot{x} & \ddot{y} - 2\dot{x}\dot{\theta} & 2\dot{y}\dot{\theta} + \ddot{x} \\ \kappa\ddot{y} & 2\dot{y}\dot{\theta} + \ddot{x} & 2\dot{x}\dot{\theta} - \ddot{y} \\ \iota\ddot{\theta} & \frac{1}{2}(\dot{x}^2 - \dot{y}^2) & -\dot{x}\dot{y} \end{bmatrix} \begin{bmatrix} 1 \\ \lambda\sin 2\theta \\ \lambda\cos 2\theta \end{bmatrix} = \begin{bmatrix} 2\cos(\phi+\theta) \\ 2\sin(\phi+\theta) \\ -a\sin\phi \end{bmatrix} f. \tag{1}$$

Appending the subscript i to the variables when referring to the dynamics of the ith agent in a multiagent system, (1) can be rewritten as

$$\begin{bmatrix} \kappa + \lambda\cos 2\theta_i & \lambda\sin 2\theta_i & 0 \\ \lambda\sin 2\theta_i & \kappa - \lambda\cos 2\theta_i & 0 \\ 0 & 0 & \iota \end{bmatrix} \begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \\ \ddot{\theta}_i \end{bmatrix} + \begin{bmatrix} 2\lambda\dot{y}_i\dot{\theta}_i\cos 2\theta_i - 2\lambda\dot{x}_i\dot{\theta}_i\sin 2\theta_i \\ 2\lambda\dot{x}_i\dot{\theta}_i\cos 2\theta_i + 2\lambda\dot{y}_i\dot{\theta}_i\sin 2\theta_i \\ \frac{1}{2}\lambda(\dot{x}_i^2 - \dot{y}_i^2)\sin 2\theta_i - \lambda\dot{x}_i\dot{y}_i\cos 2\theta_i \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \\ 0 \end{bmatrix} u_{i1} + \begin{bmatrix} -\sin\theta_i \\ \cos\theta_i \\ -\frac{a}{2} \end{bmatrix} u_{i2}. \tag{2}$$

As the main interest in controlling an agent is in the position, we set the control inputs u_{i1} and u_{i2} as

$$u_{i1} = f_{i1}\cos\theta_i + f_{i2}\sin\theta_i \quad \text{and} \quad u_{i2} = -f_{i1}\sin\theta_i + f_{i2}\cos\theta_i,$$

respectively, and the right hand side of the dynamics equation becomes

$$\begin{bmatrix} \cos\theta_i \\ \sin\theta_i \\ 0 \end{bmatrix} u_{i1} + \begin{bmatrix} -\sin\theta_i \\ \cos\theta_i \\ -\frac{a}{2} \end{bmatrix} u_{i2} = \begin{bmatrix} \cos\theta_i(f_{i1}\cos\theta_i + f_{i2}\sin\theta_i) \\ \sin\theta_i(f_{i1}\cos\theta_i + f_{i2}\sin\theta_i) \\ 0 \end{bmatrix} + \begin{bmatrix} -\sin\theta_i(-f_{i1}\sin\theta_i + f_{i2}\cos\theta_i) \\ \cos\theta_i(-f_{i1}\sin\theta_i + f_{i2}\cos\theta_i) \\ -\frac{a}{2}(-f_{i1}\sin\theta_i + f_{i2}\cos\theta_i) \end{bmatrix}$$

$$= \begin{bmatrix} f_{i1} \\ f_{i2} \\ \frac{a}{2}(f_{i1}\sin\theta_i - f_{i2}\cos\theta_i) \end{bmatrix}. \tag{3}$$

2.2. Feedback transformation and position control

We now apply a feedback transformation to the equations of motion obtained above to realize a double-integrator model for the controlled position of an individual agent. In terms of the components f_{i1} and f_{i2} of the force f with respect to the spatially fixed frame shown in Fig. 1, the upper two components of (3) may be replaced as

$$\begin{aligned} f_{i1} &= 2\lambda\dot{y}_i\dot{\theta}_i\cos 2\theta_i - 2\lambda\dot{x}_i\dot{\theta}_i\sin 2\theta_i + g_{i1}, \\ f_{i2} &= 2\lambda\dot{x}_i\dot{\theta}_i\cos 2\theta_i + 2\lambda\dot{y}_i\dot{\theta}_i\sin 2\theta_i + g_{i2}. \end{aligned} \tag{4}$$

The resulting closed loop equation then takes the form

$$\begin{bmatrix} \kappa + \lambda\cos 2\theta_i & \lambda\sin 2\theta_i & 0 \\ \lambda\sin 2\theta_i & \kappa - \lambda\cos 2\theta_i & 0 \\ 0 & 0 & \iota \end{bmatrix} \begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \\ \ddot{\theta}_i \end{bmatrix}$$

$$= \begin{bmatrix} g_{i1} \\ g_{i2} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{a}{2}(f_{i1}\sin\theta_i - f_{i2}\cos\theta_i) \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ \frac{1}{2}\lambda(\dot{x}^2 - \dot{y}^2)\sin 2\theta_i - \lambda\dot{x}_i\dot{y}_i\cos 2\theta_i \end{bmatrix}.$$

Setting

$$q_i := \begin{bmatrix} x_i \\ y_i \end{bmatrix}, \qquad M_i := \begin{bmatrix} \kappa + \lambda\cos 2\theta_i & \lambda\sin 2\theta_i \\ \lambda\sin 2\theta_i & \kappa - \lambda\cos 2\theta_i \end{bmatrix} \quad \text{and} \quad g_i := \begin{bmatrix} g_{i1} \\ g_{i2} \end{bmatrix}$$

yields the matrix equation

$$M_i\ddot{q}_i = g_i \tag{5}$$

that governs the position of the ith agent, with the orientation of the latter evolving according to

$$\iota\ddot{\theta}_i = \frac{a}{2}(f_{i1}\sin\theta_i - f_{i2}\cos\theta_i) + \frac{1}{2}\left(\dot{y}_i^2 - \dot{x}_i^2\right)\lambda\sin 2\theta_i + \dot{x}_i\dot{y}_i\lambda\cos 2\theta_i. \tag{6}$$

Note that M_i is a positive definite matrix since its eigenvalues are located at $\kappa + \lambda$ and $\kappa - \lambda$.

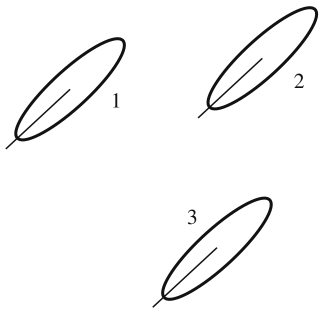

Fig. 2. A triangular school comprising three vehicles.

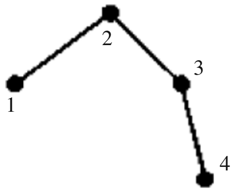

Fig. 3. Example communication structure of a four-agent system.

3. Multiagent coordination: Model reference control laws

3.1. Problem formulation

The coordination of several vehicles like that depicted in Fig. 1 to form a maneuvering school such as that shown in Fig. 2 corresponds to the stabilization of the quantities

$$x_i - x_j, \qquad y_i - y_j, \qquad \theta_i - \theta_j$$

for all index pairs (i, j) identifying distinct pairs of vehicles within the school, although the stabilization of certain coordinate pairs will follow from the stabilization of others. The organization of three vehicles, for example, requires the stabilization of the quantities

$$x_1 - x_2, \qquad x_2 - x_3, \qquad y_1 - y_2, \qquad y_2 - y_3$$

to nonzero constant values and the stabilization of the quantities

$$\theta_1 - \theta_2, \qquad \theta_2 - \theta_3$$

to zero, guaranteeing the appropriate behavior of $x_1 - x_3$, $y_1 - y_3$, and $\theta_1 - \theta_3$, although in the present work this requirement for $\theta_i - \theta_j$ is relaxed. Instead of trajectory following, which requires cumbersome algebraic manipulation to solve for the control input, the system is controlled to follow the model output trajectory, given reference input r. The model does not include rotation.

3.2. Position coordination of a network of vehicles

Let the model for the *ith* vehicle be defined as

$$\ddot{\hat{q}}_i = \Delta\hat{q}_i - \dot{\hat{q}}_i + r_i,$$

where the model is set to have the same network structure as the system, and the Laplacian operator Δ is defined depending on the communication structure, yielding, for example,

$$\Delta\hat{q}_3 = \begin{bmatrix} \hat{x}_2 - \hat{x}_3 + \hat{x}_4 - \hat{x}_3 \\ \hat{y}_2 - \hat{y}_3 + \hat{y}_4 - \hat{y}_3 \end{bmatrix}$$

for agent 3 for the structure depicted in Fig. 3. The Laplacian operator Δ is introduced in the reference model so as to mimic the behavior of the wave equation [11]. For this model represented as a PdE, an analogy can be made with a hyperbolic PDE representing an undamped string vibration.

As mentioned in the introduction, in this work we assume that the reference inputs for the reference models are specified. Moreover, the reference input is assumed to be specified for only a few of the agents. This assumption is based on the notion that only some of the agents could be equipped with instrumentation that would enable them determine their desired spatiotemporal evolution and reference inputs, by back-calculation. The reference input for the remaining agents is assumed to be zero. With this choice of reference inputs, the reference model can be induced to evolve in a fashion resembling the evolution of a hyperbolic PDE under boundary control (for details see [8]). The control law is specified so that the agent profile tracks the reference model.

For the ith agent, we propose the following control input g_i in (5):

$$g_i = -\Delta e_i + M_i \dot{e}_i + \frac{1}{2}\dot{M}_i(e_i + \dot{e}_i) + k_1 e_i + k_2 \dot{e}_i + M_i \Delta \hat{q}_i - M_i \dot{q}_i + M_i r_i, \tag{7}$$

where $e_i = \hat{q}_i - q_i$ and $k_1 > (\kappa + \lambda)$, $k_2 > 0$ are gains. Hence, it follows that

$$\begin{aligned} \ddot{e}_i &= \Delta\hat{q}_i - \dot{\hat{q}}_i + r_i - M_i^{-1}\left(\Delta q_i + M_i\dot{e}_i + \frac{1}{2}\dot{M}_i\left(e_i + \dot{e}_i\right) + k_1 e_i + k_2\dot{e}_i + M_i\left(I - M_i^{-1}\right)\Delta\hat{q}_i - M_i\dot{q}_i + M_i r_i\right) \\ &= \Delta\hat{q}_i - M_i^{-1}\Delta\hat{q}_i + M_i^{-1}\Delta\hat{q}_i - M_i^{-1}\Delta q_i + r_i - 2\dot{e}_i - \frac{1}{2}M_i^{-1}\dot{M}_i\left(e_i + \dot{e}_i\right) - \left(I - M_i^{-1}\right)\Delta\hat{q}_i - M_i^{-1}\left(k_1 e_i + k_2\dot{e}_i\right) - r_i \\ &= M_i^{-1}\Delta e_i - 2\dot{e}_i - \frac{1}{2}M_i^{-1}\dot{M}_i\left(e_i + \dot{e}_i\right) - M_i^{-1}\left(k_1 e_i + k_2\dot{e}_i\right). \end{aligned} \tag{8}$$

Since M_i is a function of θ_i, so is $\ddot{e}_i$.

In this paragraph we will define new variables so that we can rewrite the orientation equation (6) and the error equation (8), derived for individual agents, as two equations that represent the orientation and error dynamics of the whole network of agents. Henceforth, $\hat{q} = \left[\hat{q}_1^{\mathrm{T}}, \hat{q}_2^{\mathrm{T}} \dots, \hat{q}_n^{\mathrm{T}}\right]^{\mathrm{T}}$, where $\hat{q}_i^{\mathrm{T}} = \left[\hat{x}_i, \hat{y}_i\right]$, and $q = \left[q_1^{\mathrm{T}}, q_2^{\mathrm{T}} \dots, q_n^{\mathrm{T}}\right]^{\mathrm{T}}$, where $q_i^{\mathrm{T}} = [x_i, y_i]$. The error in position is given as $e = \hat{q} - q$ and $\Delta e = \left[\Delta e_1^{\mathrm{T}}, \Delta e_2^{\mathrm{T}} \dots, \Delta e_n^{\mathrm{T}}\right]^{\mathrm{T}}$. The matrix M is a block diagonal matrix of the blocks $\{M_i, i = 1 \dots n\}$ corresponding to n agents. r is the reference signal vector of suitable dimension provided to the model. g is the control signal of suitable dimension applied to the plant. T is a diagonal matrix with the diagonal elements being ι. The angular variable $\theta = [\theta_1, \theta_2 \dots, \theta_n]^{\mathrm{T}}$. Define $f_1 = [f_{11}, f_{21} \dots, f_{n1}]^{\mathrm{T}}$ and $f_2 = [f_{12}, f_{22} \dots, f_{n2}]^{\mathrm{T}}$. For the vector θ define $\sin\theta$ and $\cos\theta$ componentwise. Define $h_1(q) = \left[\dot{y}_1^2 - \dot{x}_1^2, \dot{y}_2^2 - \dot{x}_2^2 \dots, \dot{y}_n^2 - \dot{x}_n^2\right]^{\mathrm{T}}$, $h_2(q) = [\dot{x}_1\dot{y}_1, \dot{x}_2\dot{y}_2 \dots, \dot{x}_n\dot{y}_n]^{\mathrm{T}}$. Then we can combine (6) and (8) for all the agents and get the following equations:

$$\begin{aligned} \dot{e} &= \ddot{e}, \\ \ddot{e} &= M^{-1}\Delta e - 2\dot{e} - \frac{1}{2}M^{-1}\dot{M}\left(e + \dot{e}\right) - M^{-1}\left(k_1 e + k_2\dot{e}\right), \end{aligned} \tag{9}$$

$$T\ddot{\theta} = \frac{a}{2}(f_1 \sin\theta - f_2\cos\theta) + \frac{1}{2}h_1(q)\,\lambda\sin 2\theta + h_2(q)\,\lambda\cos 2\theta. \tag{10}$$

Consider the coupled nonlinear system of equations (9) and (10). In three steps we will establish that the solution to these equations exists for all time. In step 1 we will show that the right hand side of (9) and (10) is locally Lipschitz over the whole state space. In step 2 we will show that along the trajectory of the system, $\|e\|$ decreases. In step 3, we will establish the existence of a solution for all time.

Step 1 Consider the Eqs. (9) and (10). The equations can be represented in state space form using the state vector $Y = \left[e^{\mathrm{T}}, \dot{e}^{\mathrm{T}}, \theta^{\mathrm{T}}, \dot{\theta}^{\mathrm{T}}\right]^{\mathrm{T}}$. Noting that M^{-1} is a smooth function of θ, it is easy to deduce that the right hand side of (9) is a smooth function of the state variable Y and does not depend on time explicitly. Hence the partial derivatives of the function on the right hand side with respect to the states exist, and are continuous and bounded in a region in which the states themselves are bounded. Making suitable substitutions we can rewrite (10) as

$$\begin{aligned} \dot{\theta} &= \ddot{\theta}, \\ \ddot{\theta} &= F(Y) + G\left(Y, \dot{\hat{q}}, \Delta\hat{q}, r\right), \end{aligned} \tag{11}$$

where F, G are smooth functions. Note that in forming G we have used $\dot{q} = \dot{\hat{q}} - \dot{e}$. We assume that $\dot{\hat{q}}$, $\Delta\hat{q}$, r are all bounded. These assumptions are not restrictive since $\dot{\hat{q}}$ bounded implies that the desired velocity of the agents has some arbitrary upper bound. The bound on $\Delta\hat{q}$ implies that there is a upper bound on the desired distance between the agents. As explained in [8], the reference input r should be bounded for $\dot{\hat{q}}$, $\Delta\hat{q}$ to be bounded. Under these assumptions the right hand side of (11) is a smooth function of Y and has a bounded dependence on time. Hence it has continuous partial derivatives with respect to the states that remain bounded in a bounded region in the state space. Hence we can conclude that the state equation for Y has a smooth right hand side that has continuous bounded partial derivatives in any fixed bounded convex region W of the state space. Hence the right hand side of the state equation for Y is Lipschitz for all time in the region W ([15], Lem. 3.1, p. 89) and locally Lipschitz in the whole state space R^{6n} since W is arbitrary.

Step 2 Consider a trajectory starting at $Y_0 = \left(e_0, \dot{e}_0, \theta_0, \dot{\theta}_0\right) \in R^{6n}$ at time T_0. Let $[T_0, T_{\max})$ be the maximal time interval on which a solution starting at Y_0 exists. Assume $T_{\max} < \infty$ and we will get a contradiction. If $T_{\max} < \infty$, the trajectory leaves any compact subset of R^{6n} in a time $t < T_{\max}$. Consider the function defined as

$$V = \frac{1}{2}\left(e+\dot{e},\ M\left(e+\dot{e}\right)\right) - \frac{1}{2}\left(e, \Delta e\right) + \frac{1}{2}k_1\left(e, e\right) + \frac{1}{2}M\left(e, e\right) + \frac{1}{2}k_2\left(e, e\right).$$

Note that M is positive definite and $(e, \Delta e) < 0$ [8]. Then

$$\begin{aligned}
\dot{V} &= \left(e+\dot{e},\ M\left(\dot{e}+\ddot{e}\right)\right) + \frac{1}{2}\left(e+\dot{e}, \dot{M}\left(e+\dot{e}\right)\right) - \left(\dot{e}, \Delta e\right) + k_1\left(e, \dot{e}\right) + \left(e, \dot{e}\right) + k_2\left(e, \dot{e}\right) \\
&= \left(e+\dot{e}, \Delta e - M\dot{e} - \frac{1}{2}\dot{M}\left(e+\dot{e}\right) - k_1 e - k_2\dot{e}\right) + \frac{1}{2}\left(e+\dot{e}, \dot{M}\left(e+\dot{e}\right)\right) - \left(\dot{e}, \Delta e\right) + k_1\left(e, \dot{e}\right) + k_2\left(e, \dot{e}\right) \\
&= \left(e, \Delta e\right) - \left(e+\dot{e}, k_1 e + k_2\dot{e}\right) - \left(e, M\dot{e}\right) - \left(\dot{e}, M\dot{e}\right) + k_1\left(e, \dot{e}\right) + k_2\left(e, \dot{e}\right) \\
&= \left(e, \Delta e\right) - k_1\left(e, e\right) - k_2\left(\dot{e}, \dot{e}\right) - \left(\dot{e}, M\dot{e}\right) - \left(e, M\dot{e}\right) \\
&\leq \left(e, \Delta e\right) - k_1\left(e, e\right) - k_2\left(\dot{e}, \dot{e}\right) - \left(\dot{e}, M\dot{e}\right) + \left(\left(e, Me\right) + \left(\dot{e}, M\dot{e}\right)\right) \\
&= \left(e, \Delta e\right) - k_1\left(e, e\right) - k_2\left(\dot{e}, \dot{e}\right) + \left(e, Me\right).
\end{aligned}$$

The facts that $\frac{\mathrm{d}}{\mathrm{d}t}\left(e, \Delta e\right) = 2\left(\dot{e}, \Delta e\right)$ and $(e, \Delta e) < 0$ unless $e = 0$ are established in [8]. Since $k_1 > (\kappa + \lambda)$ and $(e, \Delta e) < 0$ unless $e = 0$, we have $\dot{V} < 0$ and for any $t \in (T_0, T_{\max})$ $\|e(t)\| < C\|V(T_0)\|$ and $\|\dot{e}(t)\| < C\|V(T_0)\|$.

Step 3 Consider a trajectory starting at $Y_0 = \left(e_0, \dot{e}_0, \theta_0, \dot{\theta}_0\right) \in W$ at time T_0. Define $W = \{(e, \dot{e}, \theta, \dot{\theta}) : \|e\| < 2C\|V(T_0)\|$, $\|\dot{e}\| < 2C\|V(T_0)\|$, $\|\theta\| < 2M$, $\|\dot{\theta}\| < 2M\}$ where M_1, M_2 will be specified below. Consider Eq. (6) which governs the evolution of the angular position of the ith agent. Under the assumptions that $\dot{\hat{q}}$, $\Delta\hat{q}$, r are all bounded and since $\|e\|$ and $\|\dot{e}\|$ remain bounded (Step 2), there exist constants $N_1(\|V(T_0)\|, \dot{\hat{q}}, \Delta\hat{q}, r) > 0$ and $N_2(\|V(T_0)\|, \dot{\hat{q}}, \Delta\hat{q}, r) > 0$ such that for all $T_0 < t < T_{\max}$ we have

$$\ddot{\theta}_i(t) < N_1\left|\dot{\theta}_i(t)\right| + N_2, \quad \text{given } \dot{\theta}_i(T_0). \tag{12}$$

Consider the system

$$\dot{z}(t) = N_1|z(t)| + N_2, \quad z(T_0) = \dot{\theta}_i(T_0). \tag{13}$$

The solution to this system can be bounded by

$$|z(t)| < \frac{(N_1|z(T_0)| + N_2)\, e^{N_1(t-T_0)} + N_2}{N_1} < C_{1i}, \quad \forall t : T_0 < t < T_{\max}.$$

Hence by using the comparison lemma ([15], Lem. 3.4, p. 102) for (12) and (13) we get that $\left|\dot{\theta}_i(t)\right| < C_{1i}, \forall t : T_0 < t < T_{\max}$. Hence we can conclude that there exists C_{1i} such that $|\theta_i(t)| < C_{2i}, \forall t : T_0 < t < T_{\max}$. Hence there exist constants C_1 and C_2 such that $\|\theta(t)\| < C_1$ and $\|\dot{\theta}(t)\| < C_2$ $\forall t : T_0 < t < T_{\max}$. Now choose $M = \max(C_1, C_2)$. Consider the compact set $W^c = \left\{\left(e, \dot{e}, \theta, \dot{\theta}\right) : \|e\| \leq C\|V(T_0)\|, \|\dot{e}\| \leq C\|V(T_0)\|, \|\theta\| \leq M, \|\dot{\theta}\| \leq M\right\}$. Clearly from the arguments above, the trajectory starting from Y_0 cannot leave W^c for any $t < T_{\max} < \infty$. This violates the fact that $[T_0, T_{\max})$ is the maximal time interval on which a solution starting at Y_0 exists. Hence $T_{\max} = \infty$ and a solution exists for all time for (9) and (10).

Proposition 1. *The error e converges to zero asymptotically*

Proof. Consider (9) as a time varying equation and ignore the dependence on θ. Since M is a positive definite matrix with eigenvalues located at $\kappa + \lambda$ and $\kappa - \lambda$, from Step 2 we have

$$\begin{aligned}
V(e, \dot{e}, t) &\leq \frac{\kappa + \lambda}{2}\left(e+\dot{e},\ \left(e+\dot{e}\right)\right) - \frac{1}{2}\left(e, \Delta e\right) + \frac{1}{2}k_1\left(e, e\right) + \frac{1}{2}M\left(e, e\right) + \frac{1}{2}k_2\left(e, e\right) = W_1(e, \dot{e}), \\
V(e, \dot{e}, t) &\geq -\frac{1}{2}\left(e, \Delta e\right) + \frac{1}{2}k_1\left(e, e\right) + \frac{1}{2}M\left(e, e\right) + \frac{1}{2}k_2\left(e, e\right) = W_2(e, \dot{e}), \\
\dot{V}(e, \dot{e}, t) &\leq -k_1\left(e, e\right) - k_2\left(\dot{e}, \dot{e}\right) = W_3(e, \dot{e}).
\end{aligned}$$

Hence we obtain (from [15], Th. 4.8, p. 151) that $\|e\| \to 0$ as $t \to \infty$. □

Remark 1. The model can be modified depending on the desired response, for instance, as

$$\ddot{\hat{q}} = k_L\Delta\hat{q} - k_C\dot{\hat{q}} + r$$

where positive constant k_L scales the Laplacian operator, i.e. the tension of a spring, and positive constant k_C scales the viscous damping. In this case, the proper control g is

$$g = \Delta q + M\dot{e} + \frac{1}{2}\dot{M}\left(e+\dot{e}\right) + k_1 e + k_2\dot{e} + M\left(k_L I - M^{-1}\right)\Delta\hat{q} - Mk_C\dot{\hat{q}} + Mr.$$

with sufficiently large gains k_1, k_2.

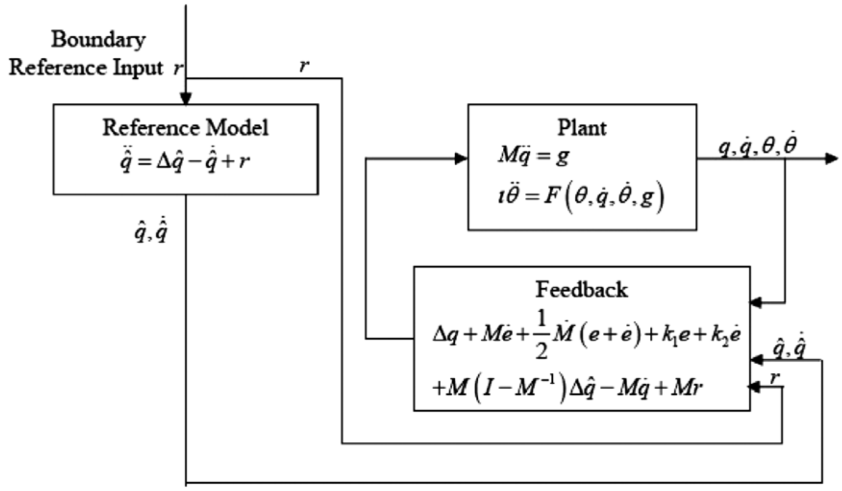

Fig. 4. Implementation schematics of the MRC law.

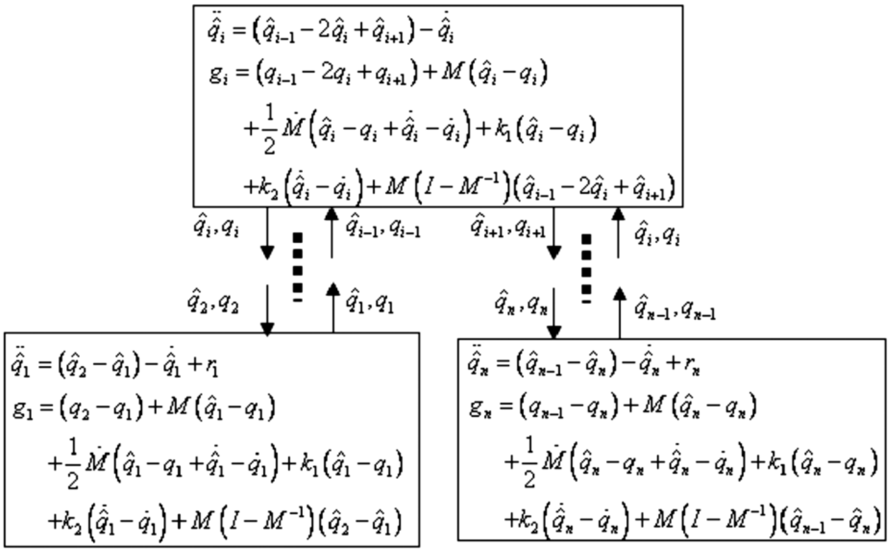

Fig. 5. Localized computation of the reference trajectory and control signal.

Remark 2. Collision avoidance can be added using potential function type terms as shown in [7], i.e. introducing a potential function U into both the model and the plant as follows:

$$\ddot{\hat{q}} = \Delta\hat{q} - \dot{\hat{q}} - U\left(q\left(\cdot, t\right)\right) + r,$$
$$\ddot{q} = \Delta q + r,$$
$$h = g - MU\left(q\left(\cdot, t\right)\right),$$

where g is defined as (7). Notice that the argument of U in the model equation is the plant state, rather than the model state, for producing the same error equation as (8).

Fig. 4 shows the implementation schematics for the MRC law developed in this section. Fig. 5 shows the localized nature of the computations involved for a case of n agents in which agents 1 and n are provided the reference inputs. The ith agent communicates only with the $(i-1)$ th and the $(i-1)$ th agents. Agents $\{2, \ldots, n-1\}$ move solely based on the inputs from their neighbors. As seen, all the computations required for obtaining the control signal are performed locally by each agent.

3.3. Orientation behavior during position coordination

Given the control for position tracking it may be necessary to have a qualitative estimate of the evolution of θ, such as boundedness of angular velocity and steady state properties.

3.3.1. Boundedness of rotational acceleration

Consider

$$\iota\ddot{\theta}_i = -\frac{1}{2}\lambda\left(\dot{x}_i^2 - \dot{y}_i^2\right)\sin 2\theta_i + \lambda\dot{x}_i\dot{y}_i\cos 2\theta_i + \frac{a}{2}\left(f_{i1}\sin\theta_i - f_{i2}\cos\theta_i\right)$$

where

$$\begin{aligned} f_{i1} &= 2\lambda\dot{y}_i\dot{\theta}_i\cos 2\theta_i - 2\lambda\dot{x}_i\dot{\theta}_i\sin 2\theta_i + g_{i1}, \\ f_{i2} &= 2\lambda\dot{x}_i\dot{\theta}_i\cos 2\theta_i + 2\lambda\dot{y}_i\dot{\theta}_i\sin 2\theta_i + g_{i2}. \end{aligned}$$

Define

$$Q_i = 2\lambda\dot{\theta}_i\begin{bmatrix} -\sin 2\theta_i & \cos 2\theta_i \\ \cos 2\theta_i & \sin 2\theta_i \end{bmatrix}, \qquad f_i = \begin{bmatrix} f_{i1} \\ f_{i2} \end{bmatrix}, \quad \text{and} \quad f_i = Q_i\dot{q}_i + g_i.$$

Then

$$\begin{aligned} \iota\ddot{\theta}_i &= -\frac{1}{2}\lambda\left(\dot{x}_i^2 - \dot{y}_i^2\right)\sin 2\theta_i + \lambda\dot{x}_i\dot{y}_i\cos 2\theta_i + \frac{a}{2}\left[\sin\theta_i \quad -\cos\theta_i\right] f_i \\ &= \frac{\lambda}{2}\dot{q}_i^{\mathrm{T}}\begin{bmatrix} -\sin 2\theta_i & \cos 2\theta_i \\ \cos 2\theta_i & \sin 2\theta_i \end{bmatrix}\dot{q}_i \\ &\quad + \frac{a}{2}\left[\sin\theta_i \quad -\cos\theta_i\right]\left(Q_i\dot{q}_i + \Delta q_i + M_i\dot{e}_i + \frac{1}{2}\dot{M}_i\left(e_i + \dot{e}_i\right) + k_1 e_i + k_2\dot{e}_i + \left(M_i - I\right)\Delta\hat{q}_i - M\dot{q}_i + M_i r_i\right) \\ &= \frac{\lambda}{2}\dot{q}_i^{\mathrm{T}}\begin{bmatrix} -\sin 2\theta_i & \cos 2\theta_i \\ \cos 2\theta_i & \sin 2\theta_i \end{bmatrix}\dot{q}_i \\ &\quad + \frac{a}{2}\left[\sin\theta_i \quad -\cos\theta_i\right]\left(-Q_i\dot{e}_i + Q_i\dot{\hat{q}}_i - \Delta e_i + M_i\dot{e}_i + \frac{1}{2}\dot{M}_i\left(e_i + \dot{e}_i\right) + k_1 e_i + k_2\dot{e}_i + M_i\Delta\hat{q}_i - M\dot{q}_i + M_i r_i\right). \end{aligned}$$

At the steady state, i.e. $e = 0$,

$$\begin{aligned} \iota\ddot{\theta}_i &= \frac{\lambda}{2}\dot{q}_i^{\mathrm{T}}\begin{bmatrix} -\sin 2\theta_i & \cos 2\theta_i \\ \cos 2\theta_i & \sin 2\theta_i \end{bmatrix}\dot{q}_i + \frac{a}{2}\left[\sin\theta_i \quad -\cos\theta_i\right]\left(Q_i\dot{\hat{q}}_i + M_i\Delta\hat{q}_i - M\dot{\hat{q}} + M_i r_i\right) \\ &= \frac{\lambda}{2}\dot{\hat{q}}_i^{\mathrm{T}}\begin{bmatrix} -\sin 2\theta_i & \cos 2\theta_i \\ \cos 2\theta_i & \sin 2\theta_i \end{bmatrix}\dot{\hat{q}}_i + \frac{a}{2}\left[\sin\theta_i \quad -\cos\theta_i\right]\left(Q_i\dot{\hat{q}}_i + M_i\ddot{\hat{q}}_i\right). \end{aligned} \tag{14}$$

Hence, if the acceleration and velocity of the model are bounded, then the angular acceleration of the system is bounded as well.

3.3.2. Zero acceleration of the model in the steady state

In this case, (14) becomes

$$\begin{aligned} \iota\ddot{\theta}_i &= \frac{\lambda}{2}\dot{\hat{q}}_i^{\mathrm{T}}\begin{bmatrix} -\sin 2\theta_i & \cos 2\theta_i \\ \cos 2\theta_i & \sin 2\theta_i \end{bmatrix}\dot{\hat{q}}_i + \frac{a}{2}\left[\sin\theta_i \quad -\cos\theta_i\right] Q_i\dot{\hat{q}}_i \\ &= \frac{\lambda}{2}\dot{\hat{q}}_i^{\mathrm{T}}\begin{bmatrix} -\sin 2\theta_i & \cos 2\theta_i \\ \cos 2\theta_i & \sin 2\theta_i \end{bmatrix}\dot{\hat{q}}_i + \frac{a}{2}\left[\sin\theta_i \quad -\cos\theta_i\right] 2\lambda\dot{\theta}_i\begin{bmatrix} -\sin 2\theta_i & \cos 2\theta_i \\ \cos 2\theta_i & \sin 2\theta_i \end{bmatrix}\dot{\hat{q}}_i \\ &= \frac{\lambda}{2}\left[\dot{\hat{x}}_i + 2a\dot{\theta}_i\sin\theta_i \quad \dot{\hat{y}}_i - 2a\dot{\theta}_i\cos\theta_i\right]\begin{bmatrix} -\sin 2\theta_i & \cos 2\theta_i \\ \cos 2\theta_i & \sin 2\theta_i \end{bmatrix}\dot{\hat{q}}_i. \end{aligned}$$

Let a vehicle be aligned with the model trajectory and its angular velocity be zero, i.e.

$$\dot{\theta}_i = 0, \quad \tan\theta_i = \frac{\dot{\hat{y}}_i}{\dot{\hat{x}}_i}.$$

Then, the resulting $\ddot{\theta}_i = 0$, which means that once aligned, a vehicle continues to move with the appropriate orientation.

3.3.3. Linear motion of the model under attained tracking

Using the linear motion assumption of the model, set

$$\begin{aligned} \ddot{\hat{q}}_i &= \alpha_i(t)\,\phi_i, \\ \dot{\hat{q}}_i &= \beta_i(t)\,\phi_i. \end{aligned}$$

Then, (14) becomes

$$\begin{aligned}
\iota\ddot{\theta}_i &= \frac{\lambda}{2}\dot{\hat{q}}_i^{\mathrm{T}}\begin{bmatrix}-\sin 2\theta_i & \cos 2\theta_i\\ \cos 2\theta_i & \sin 2\theta_i\end{bmatrix}\dot{\hat{q}}_i + \frac{a}{2}\begin{bmatrix}\sin\theta_i & -\cos\theta_i\end{bmatrix}\left(Q_i\dot{\hat{q}}_i + M_i\ddot{\hat{q}}_i\right)\\
&= \frac{\lambda}{2}\beta_i^2\phi_i^{\mathrm{T}}\begin{bmatrix}-\sin 2\theta_i & \cos 2\theta_i\\ \cos 2\theta_i & \sin 2\theta_i\end{bmatrix}\phi_i\\
&\quad + \frac{a}{2}\begin{bmatrix}\sin\theta_i & -\cos\theta_i\end{bmatrix}\left(2\lambda\beta_i\dot{\theta}_i\begin{bmatrix}-\sin 2\theta_i & \cos 2\theta_i\\ \cos 2\theta_i & \sin 2\theta_i\end{bmatrix}\phi_i + \alpha_i\begin{bmatrix}\kappa+\lambda\cos 2\theta_i & \lambda\sin 2\theta_i\\ \lambda\sin 2\theta_i & \kappa-\lambda\cos 2\theta_i\end{bmatrix}\phi_i\right).
\end{aligned}$$

Suppose a vehicle is aligned with the model trajectory, and the angular velocity of the vehicle is zero, i.e.

$$\dot{\theta}_i = 0, \qquad \phi_i = \begin{bmatrix}\cos\theta_i\\ \sin\theta_i\end{bmatrix}.$$

Then

$$\begin{aligned}
\iota\ddot{\theta}_i &= \frac{\lambda}{2}\beta_i^2\phi_i^{\mathrm{T}}\begin{bmatrix}-\sin 2\theta_i & \cos 2\theta_i\\ \cos 2\theta_i & \sin 2\theta_i\end{bmatrix}\phi_i + \frac{a}{2}\begin{bmatrix}\sin\theta_i & -\cos\theta_i\end{bmatrix}\left(\alpha_i\begin{bmatrix}\kappa+\lambda\cos 2\theta_i & \lambda\sin 2\theta_i\\ \lambda\sin 2\theta_i & \kappa-\lambda\cos 2\theta_i\end{bmatrix}\phi_i\right)\\
&= 0.
\end{aligned}$$

That is, once aligned, the vehicle will not undergo unnecessary orientation change.

4. Simulation

The system under consideration consists of four vehicles with the communication structure shown in Fig. 3. A suitable reference input is assumed to be given to agents 1 and 4, which can be considered as boundary agents. The reference input for agents 2 and 3 is 0. Their motion is dictated by the communication structure.

4.1. Model tracking

In this section we demonstrate the performance of the control law illustrated in Fig. 4. The control objective is to make the plant follow the model. The reference model is chosen so that its motion under the influence of boundary inputs resembles that of a string under boundary inputs [8]. The reference positions are indicated as $\hat{q}_i = [x_{ri}, y_{ri}]$ and the plant position as $q_i = \left[x_{pi}, y_{pi}\right]$ where i corresponds to the agent number. The reference model is given as

$$\begin{aligned}
\ddot{\hat{q}}_1 &= \hat{q}_2 - \hat{q}_1 - \dot{\hat{q}}_1 + r_1,\\
\ddot{\hat{q}}_2 &= \hat{q}_3 - 2\hat{q}_2 + \hat{q}_1 - \dot{\hat{q}}_2 + r_2,\\
\ddot{\hat{q}}_3 &= \hat{q}_4 - 2\hat{q}_3 + \hat{q}_2 - \dot{\hat{q}}_3 + r_3,\\
\ddot{\hat{q}}_4 &= \hat{q}_3 - \hat{q}_4 - \dot{\hat{q}}_4 + r_4.
\end{aligned} \tag{15}$$

The reference inputs are specified to be

$$r_1 = \begin{bmatrix}\begin{Bmatrix}-x_{r2} + t - 1,\ 0 \le t < 2,\\ -x_{r2} + \left(1.0625t^4 - 7.5t^3 + 15t^2 - 8t + 1\right), 2 \le t < 4,\\ -x_{r2} + 1, 4 \le t < 15,\end{Bmatrix}\\ \begin{Bmatrix}-y_{r2} - 1, 0 \le t < 2,\\ -y_{r2} + \left(-1.0625t^4 + 7.5t^3 - 15t^2 + 10t - 4\right), 2 \le t < 4\\ -y_{r2} + t - 3, 4 \le t < 15\end{Bmatrix}\end{bmatrix},$$

$$r_2 = r_3 = \begin{bmatrix}0\\0\end{bmatrix},$$

$$r_4 = \begin{bmatrix}\begin{Bmatrix}-x_{r3} + t - 1, \quad 0 \le t < 2\\ -x_{r3} + \left(14.625t^5 - 142.375t^4 + 480t^3 - 660t^2 + 352t - 92\right), \quad 2 \le t < 4,\\ -x_{r2} + 4, \quad 4 \le t < 15,\end{Bmatrix}\\ \begin{Bmatrix}-y_{r3} - 4, \quad 0 \le t < 2,\\ -y_{r3} + \left(14.625t^5 - 144.5t^4 + 495t^3 - 690t^2 + 370t - 100\right), \quad 2 \le t < 4,\\ -y_{r3} + t - 3, \quad 4 \le t < 15.\end{Bmatrix}\end{bmatrix}.$$

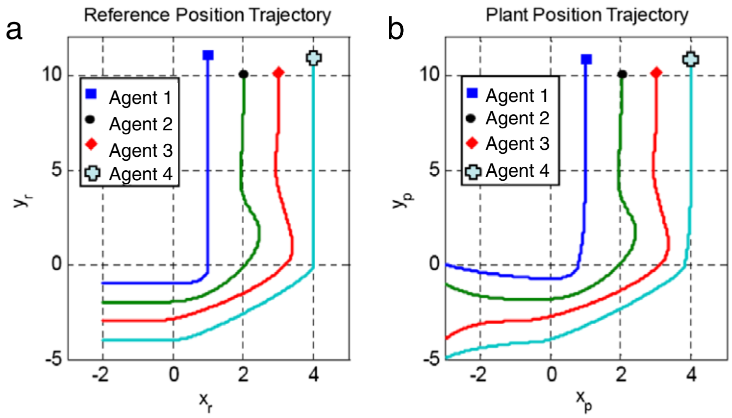

Fig. 6. Reference position profile (a) and corresponding position profile of plant (b) without disturbance.

The plant model is given as

$$\ddot{q}_i = \begin{bmatrix} \ddot{x}_{pi} \\ \ddot{y}_{pi} \end{bmatrix} = \begin{bmatrix} 2+\cos 2\theta_i & \sin 2\theta_i \\ \sin 2\theta_i & 2-\cos 2\theta_i \end{bmatrix}^{-1} \begin{bmatrix} g_{xi} \\ g_{yi} \end{bmatrix}, \quad i = 1, 2, 3, 4, \tag{16}$$

$$\begin{aligned} \ddot{\theta}_i &= \left(2\dot{y}_{pi}\dot{\theta}_i \cos 2\theta_i - 2\dot{x}_{pi}\dot{\theta}_i \sin 2\theta_i + g_{xi}\right)\sin\theta_i \\ &\quad - \left(2\dot{x}_{pi}\dot{\theta}_i \cos 2\theta_i + 2\dot{y}_{pi}\dot{\theta}_i \sin 2\theta_i + g_{yi}\right)\cos\theta_i + \frac{1}{2}\left(\dot{y}_{pi}^2 - \dot{x}_{pi}^2\right)\sin 2\theta_i + \dot{x}_{pi}\dot{y}_{pi}\cos 2\theta_i. \end{aligned}$$

The control law is given by

$$\begin{bmatrix} g_{xi} \\ g_{yi} \end{bmatrix} = \Delta q_i + M_i\dot{e}_i + \frac{1}{2}\dot{M}_i\left(e_i + \dot{e}_i\right) + 5e_i + 5\dot{e}_i + M_i\left(I - M_i^{-1}\right)\Delta\hat{q}_i + M_i r_i, \quad i = 1, 2, 3, 4 \tag{17}$$

where the following definitions apply:

$$\Delta q_1 = \begin{bmatrix} x_{p2} - x_{p1} \\ y_{p2} - y_{p1} \end{bmatrix}, \qquad \Delta q_2 = \begin{bmatrix} x_{p3} - 2x_{p2} + x_{p1} \\ y_{p3} - 2y_{p2} + y_{p1} \end{bmatrix}, \qquad \Delta q_3 = \begin{bmatrix} x_{p4} - 2x_{p3} + x_{p2} \\ y_{p4} - 2y_{p3} + y_{p2} \end{bmatrix},$$

$$\Delta q_4 = \begin{bmatrix} x_{p3} - x_{p4} \\ y_{p3} - y_{p4} \end{bmatrix}, \qquad \Delta\hat{q}_1 = \begin{bmatrix} x_{r2} - x_{r1} \\ y_{r2} - y_{r1} \end{bmatrix}, \qquad \Delta\hat{q}_2 = \begin{bmatrix} x_{r3} - 2x_{r2} + x_{r1} \\ y_{r3} - 2y_{r2} + y_{r1} \end{bmatrix},$$

$$\Delta\hat{q}_3 = \begin{bmatrix} x_{r4} - 2x_{r3} + x_{r2} \\ y_{r4} - 2y_{r3} + y_{r2} \end{bmatrix}, \qquad \Delta\hat{q}_4 = \begin{bmatrix} x_{r3} - x_{r4} \\ y_{r3} - y_{r4} \end{bmatrix}.$$

$$M_i = \begin{bmatrix} 2+\cos 2\theta_i & \sin 2\theta_i \\ \sin 2\theta_i & 2-\cos 2\theta_i \end{bmatrix}, \qquad \dot{M}_i = \begin{bmatrix} -2\sin 2\theta_i & \cos 2\theta_i \\ \cos 2\theta_i & 2\sin 2\theta_i \end{bmatrix}, \quad i = 1, 2, 3, 4,$$

$$e_i = \begin{bmatrix} x_{ri} - x_{pi} \\ y_{ri} - y_{pi} \end{bmatrix}, \qquad \dot{e}_i = \begin{bmatrix} \dot{x}_{ri} - \dot{x}_{pi} \\ \dot{y}_{ri} - \dot{y}_{pi} \end{bmatrix}, \quad i = 1, 2, 3, 4.$$

The initial position and velocity for the plant and the reference model are chosen as follows:

$$\begin{bmatrix} x_{p1} \\ y_{p1} \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} x_{p2} \\ y_{p2} \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} x_{p3} \\ y_{p3} \end{bmatrix} = \begin{bmatrix} -3 \\ -4 \end{bmatrix}, \quad \begin{bmatrix} x_{p4} \\ y_{p4} \end{bmatrix} = \begin{bmatrix} -3 \\ -5 \end{bmatrix},$$

$$\begin{bmatrix} \dot{x}_{p1} \\ \dot{y}_{p1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \dot{x}_{p2} \\ \dot{y}_{p2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \dot{x}_{p3} \\ \dot{y}_{p3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \dot{x}_{p4} \\ \dot{y}_{p4} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$\begin{bmatrix} x_{r1} \\ y_{r1} \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} x_{r2} \\ y_{r2} \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}, \quad \begin{bmatrix} x_{r3} \\ y_{r3} \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}, \quad \begin{bmatrix} x_{r4} \\ y_{r4} \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \end{bmatrix},$$

$$\begin{bmatrix} \dot{x}_{r1} \\ \dot{y}_{r1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \dot{x}_{r2} \\ \dot{y}_{r2} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \dot{x}_{r3} \\ \dot{y}_{r3} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \dot{x}_{r4} \\ \dot{y}_{r4} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Fig. 6 shows the simulated profiles that indicate that the control law (17) performs well.

Fig. 7 shows the vehicle rotation during tracking. The angular position of agent 1 settles close to $-3\pi/2$ and the angular positions of agents 2, 3, and 4 settle close to $\pi/2$. These final orientations of the agents are in the direction of motion.

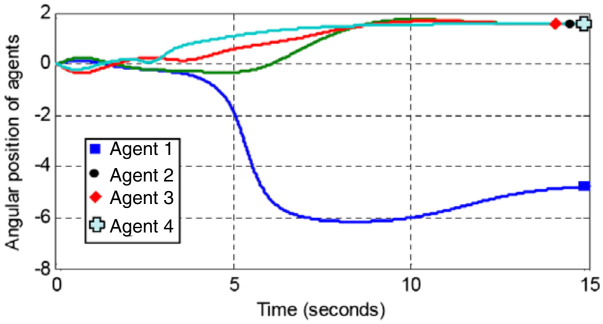

Fig. 7. Vehicle rotation change during tracking without disturbance.

5. Conclusion

On the basis of the framework of PdEs on graphs, the present work provides a control law design methodology for a network of underactuated aquatic vehicles with strongly nonlinear dynamics and tracking as the objective. Future work will address control under uncertainty.

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